

# **Analytical Solution for Cold-air-drainage flow On Sloping Forest**

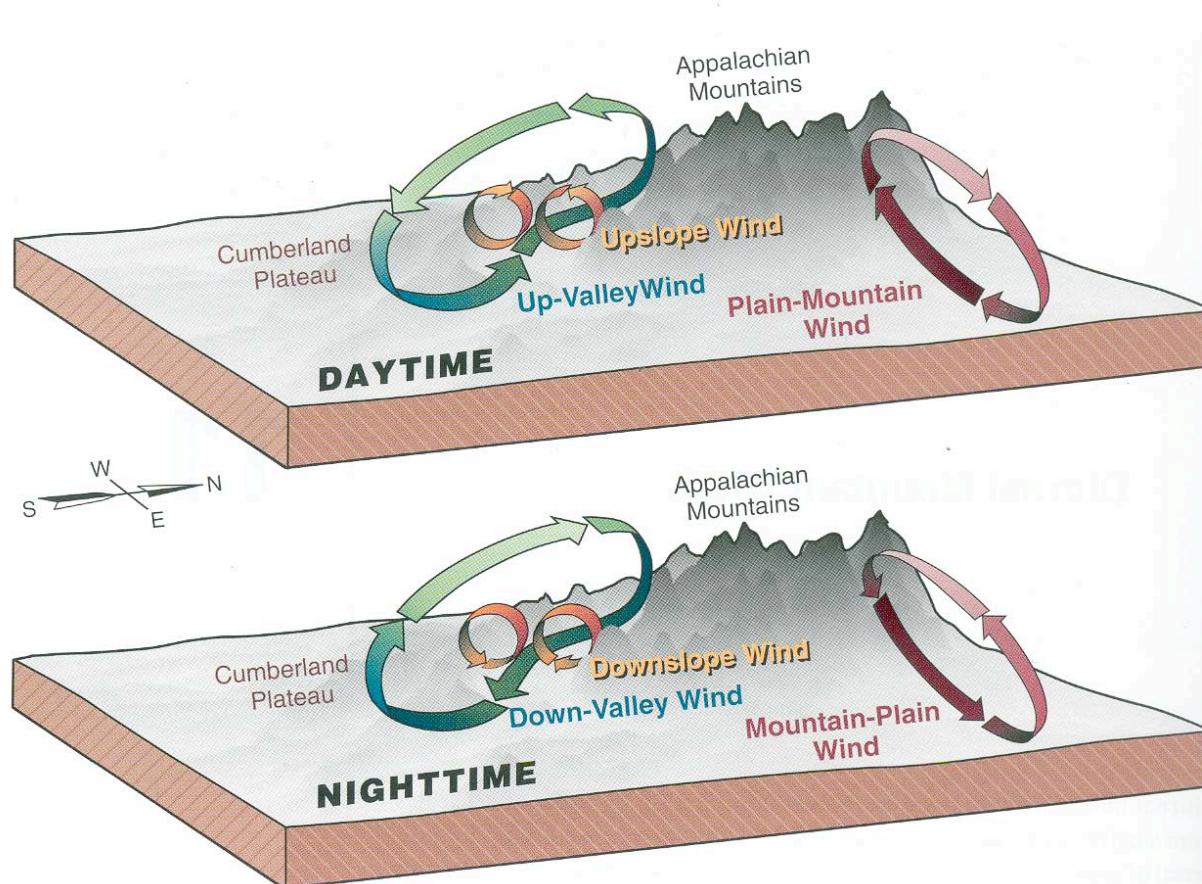
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# Cold-air-drainage Flow



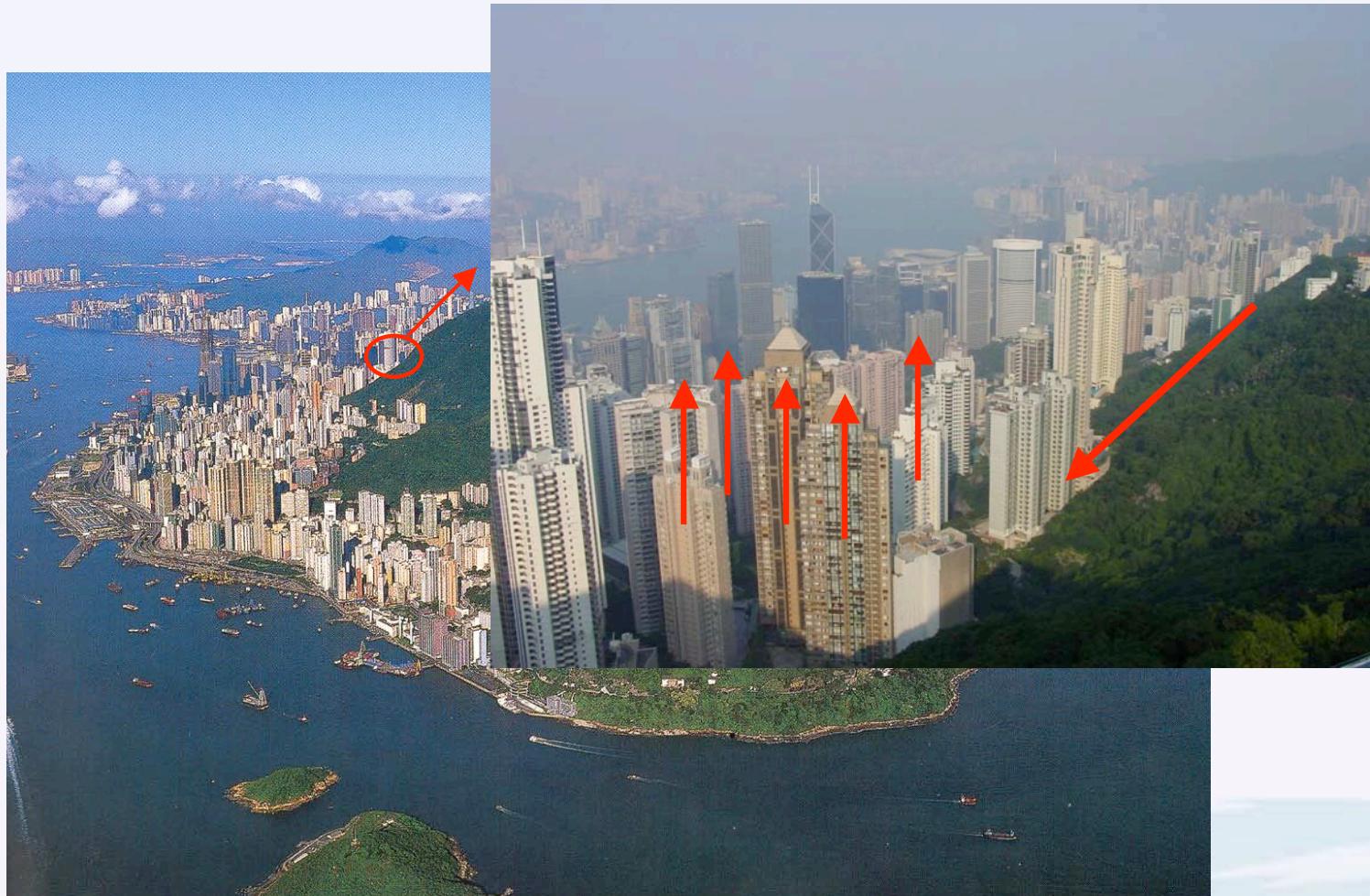
Source: C.David Whiteman "Mountain Meterology"

# Cold-air-drainage Flow

- **Mitigate the nocturnal UHI**
  - *Kitada, 1998; Ohashi and Kida, 2002*
- **Disperse the urban pollution**
  - *Baumbach and Vogt, 1999; Egan, 1984; Lu and Turco, 1994*
- **Influence the nocturnal ecosystem-atmosphere exchange**
  - *Lee and Hu, 2002; Turnipseed et al, 2003; Yi et al, 2000*

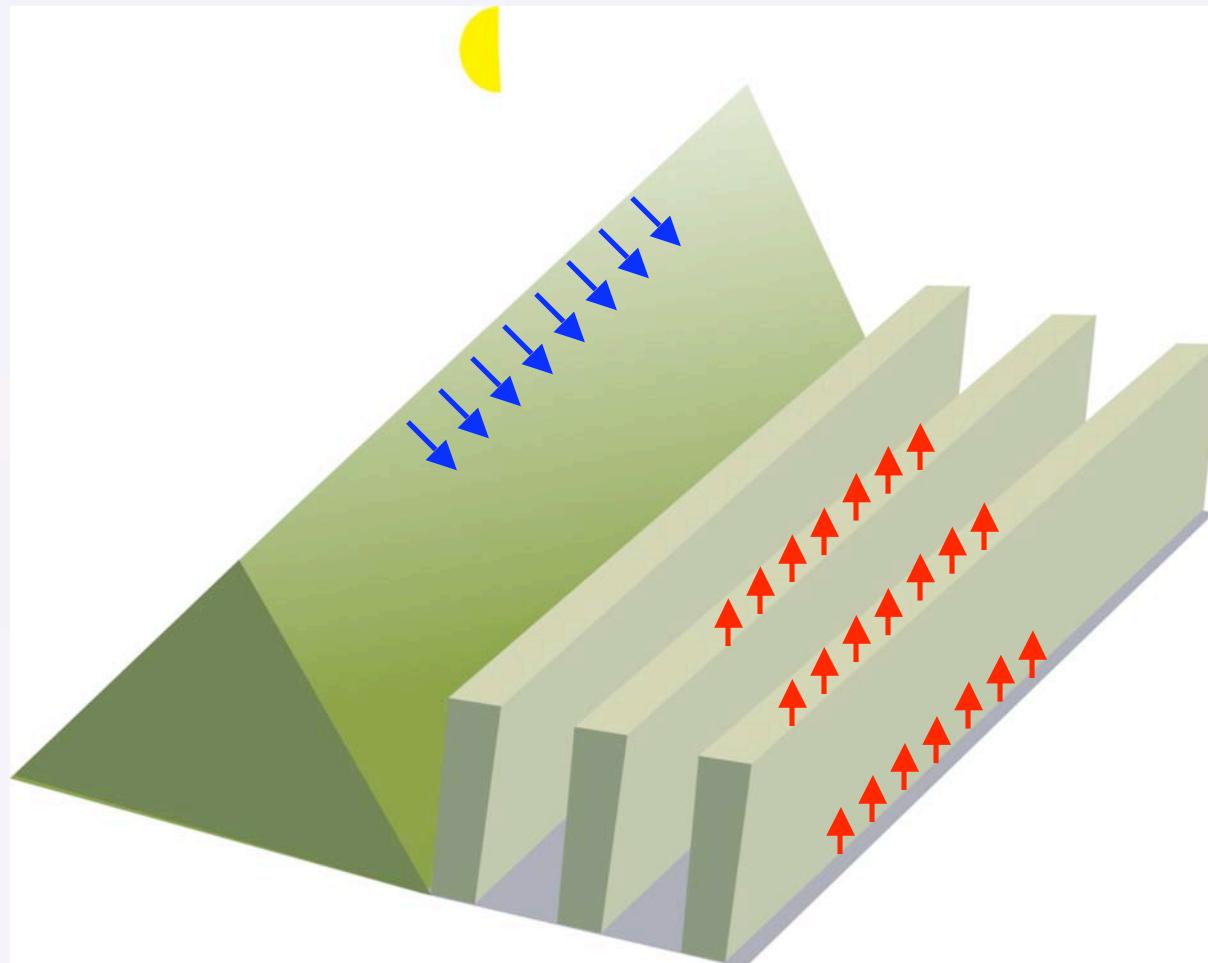


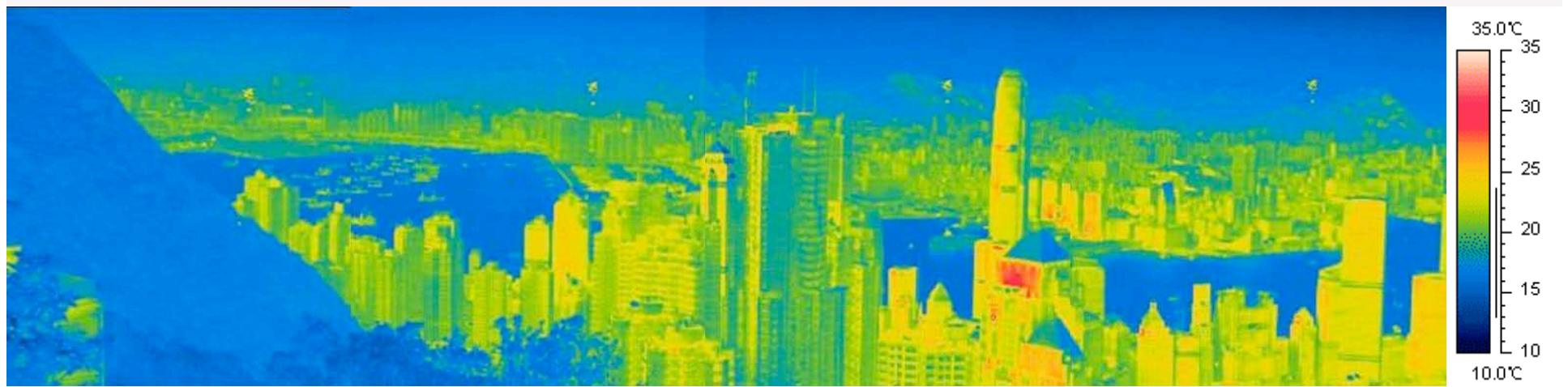
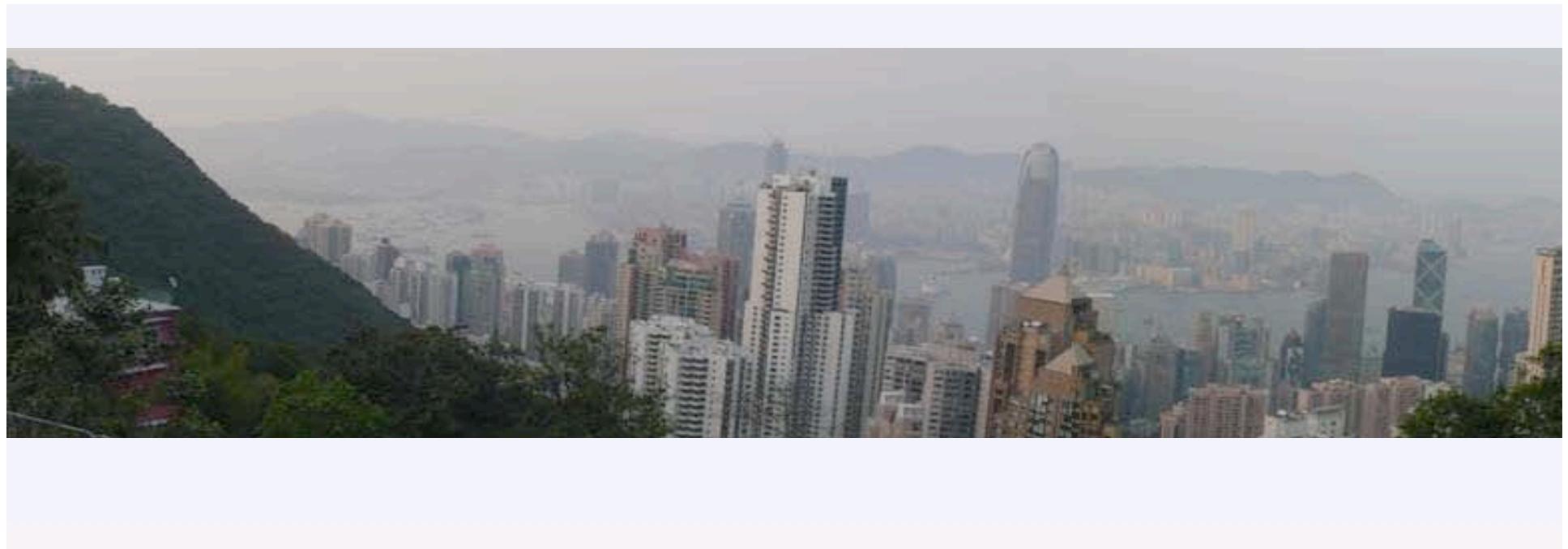
# Hong Kong Island



Source: [www.gearthblog.com](http://www.gearthblog.com)

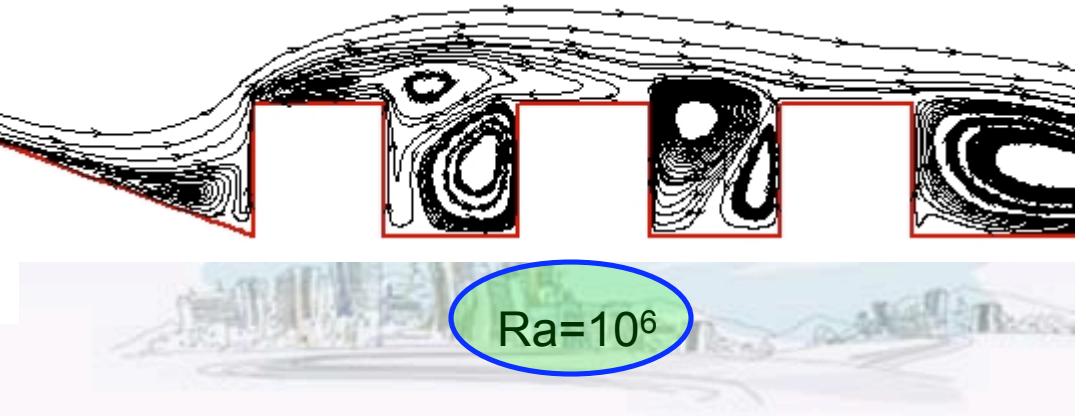
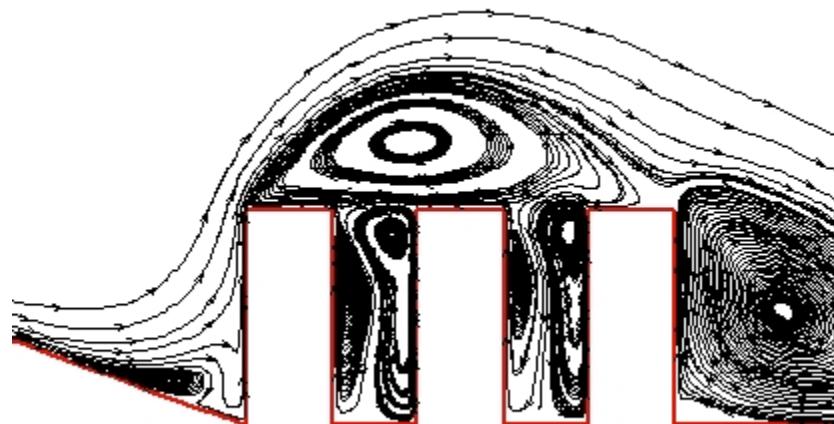
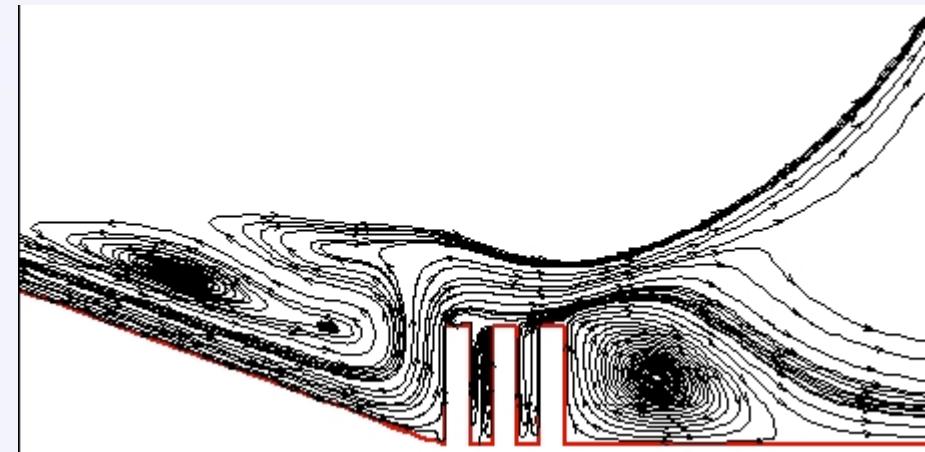
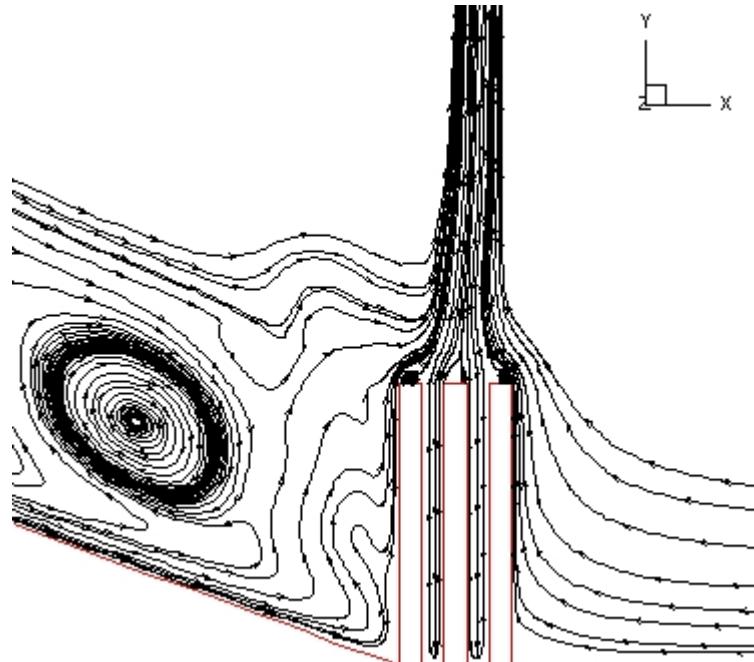
# Physical model





19:00 pm

Mar 15, 2008



# Analytical Models

- **Prandtl model**
  - One dimensional, but gives the detailed structure of the flow profile
- **Hydraulic model**
  - Only provide the layer-averaged characteristic scales of flow parameters, i.e., velocity, momentum thickness, buoyancy deficit

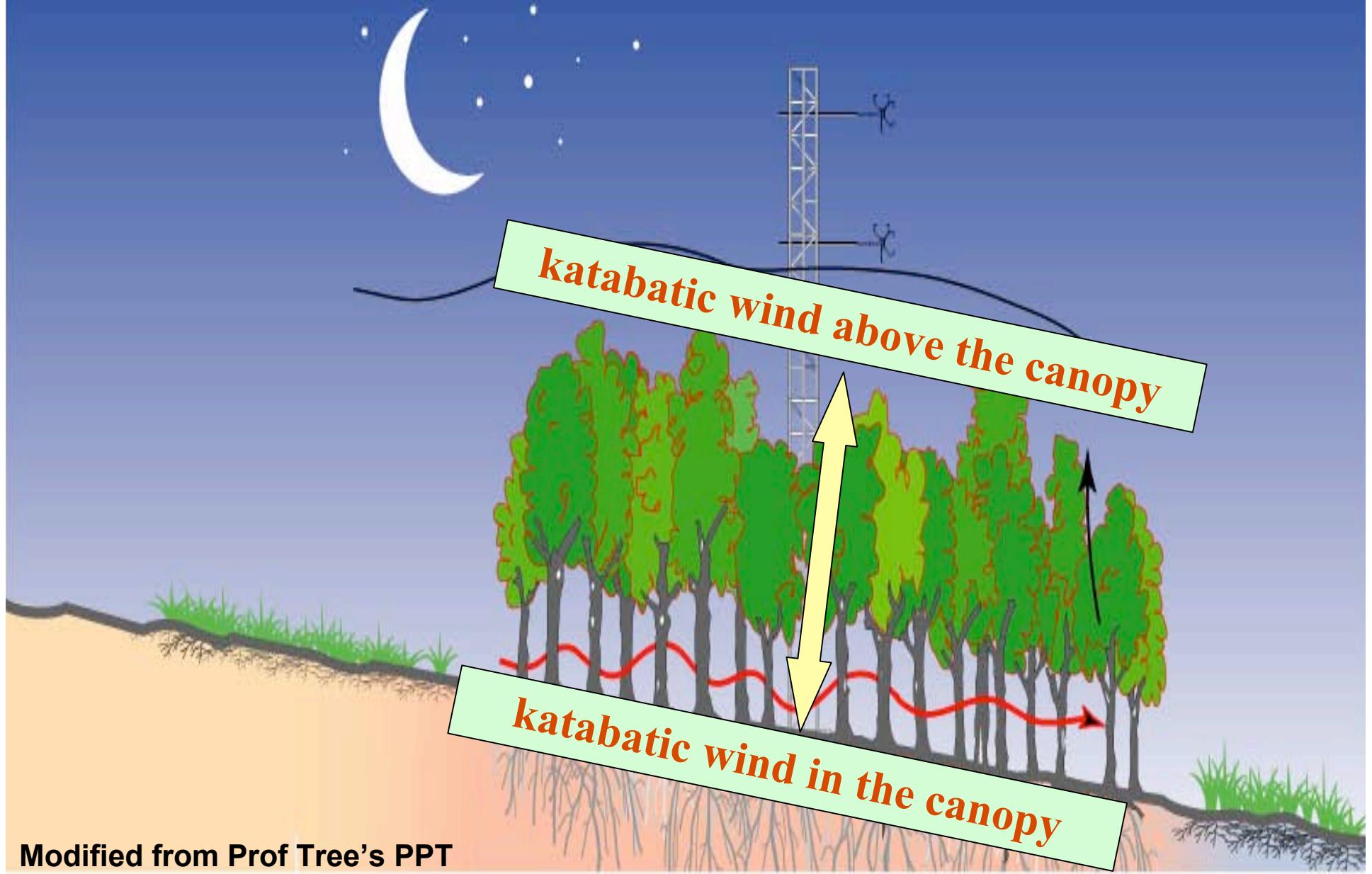


# Vegetation on Slope Flow

- Few studies address this problem
  - Bergen, 1969; Devito and Miller, 1983
- Katabatic flow can occur both within and above the tree canopies
  - Komatsu et al, 2003; Devito and miller, 1983; Pypker, 2007
- Understand the flow structure can help to estimate the surface fluxes



# Cold-air-drainage winds---Forest Canopy



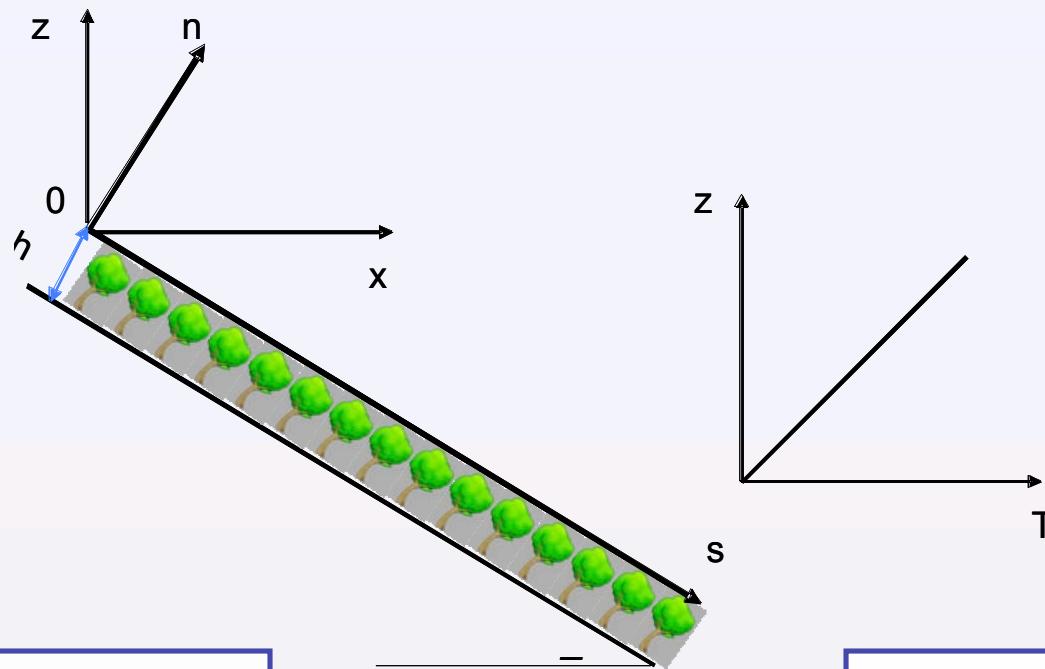
Modified from Prof Tree's PPT

# Aims

Propose a simple analytical model  
by coupling both above and within  
tree canopies



# Cold-air-drainage Flow Model



## Assumptions:

- One dimensional normal to the slope
- Constant deficit of potential temperature in the canopy
- Non-linear advection term is ignored in the momentum equation

$c_D$  --- drag coefficient;  
 $a$  ---- leaf area density;  
LAI—leaf area index;  
 $\Delta\theta$  --- deficit of the potential temperature ;  
 $h$  ---- canopy height

# Flow Within Canopy (non-uniform)

Momentum equation

$$\frac{\partial \overline{u'w'}}{\partial n} = g\beta\Delta\theta \sin \alpha + c_D a \overline{u}^2(n)$$

Parameterizing the Reynolds stress ([Yi, 2008](#)):

$$\tau(z)/\rho = -\overline{u'w'}(z) = c_D(z) \overline{u}^2(z)$$

$$\frac{\partial(c_D(n)\overline{u}^2(n))}{\partial n} = g\beta\Delta\theta \sin \alpha + c_D(n)a(n)\overline{u}^2(n)$$

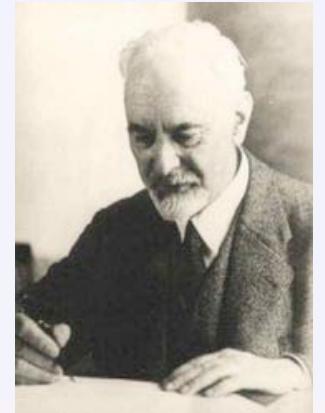
$$U(n) = -\left(\frac{c_D(0)}{c_D(n)} U^2(0) e^{-[LAI-L(n)]} - \frac{g\beta\Delta\theta \sin \alpha}{c_D(n)} \int_n^0 e^{-[L(n')-L(n)]} dn'\right)^{1/2}$$

$$L(n) = \int_{-h}^n a(n') dn' \quad LAI = L(0)$$

# Flow Above Canopy

Prandtl Model

$$\begin{cases} g\beta\Delta\theta \sin \alpha = k_m \frac{d^2 u(n)}{dn^2} \\ \gamma u(n) \sin \alpha = k_h \frac{d^2 \Delta\theta}{dn^2} \end{cases}$$



Prandtl in 1905



$$u(n) = K e^{-n/l} [\Delta\theta_s \sin(n/l) - C' \cos(n/l)]$$

$$l = \left( \frac{4k_m k_h}{N^2 \sin^2 \alpha} \right)^{\frac{1}{4}}$$

$$K = \frac{g\beta}{N} \sqrt{\frac{k_h}{k_m}}$$



# Coupling at Canopy Top

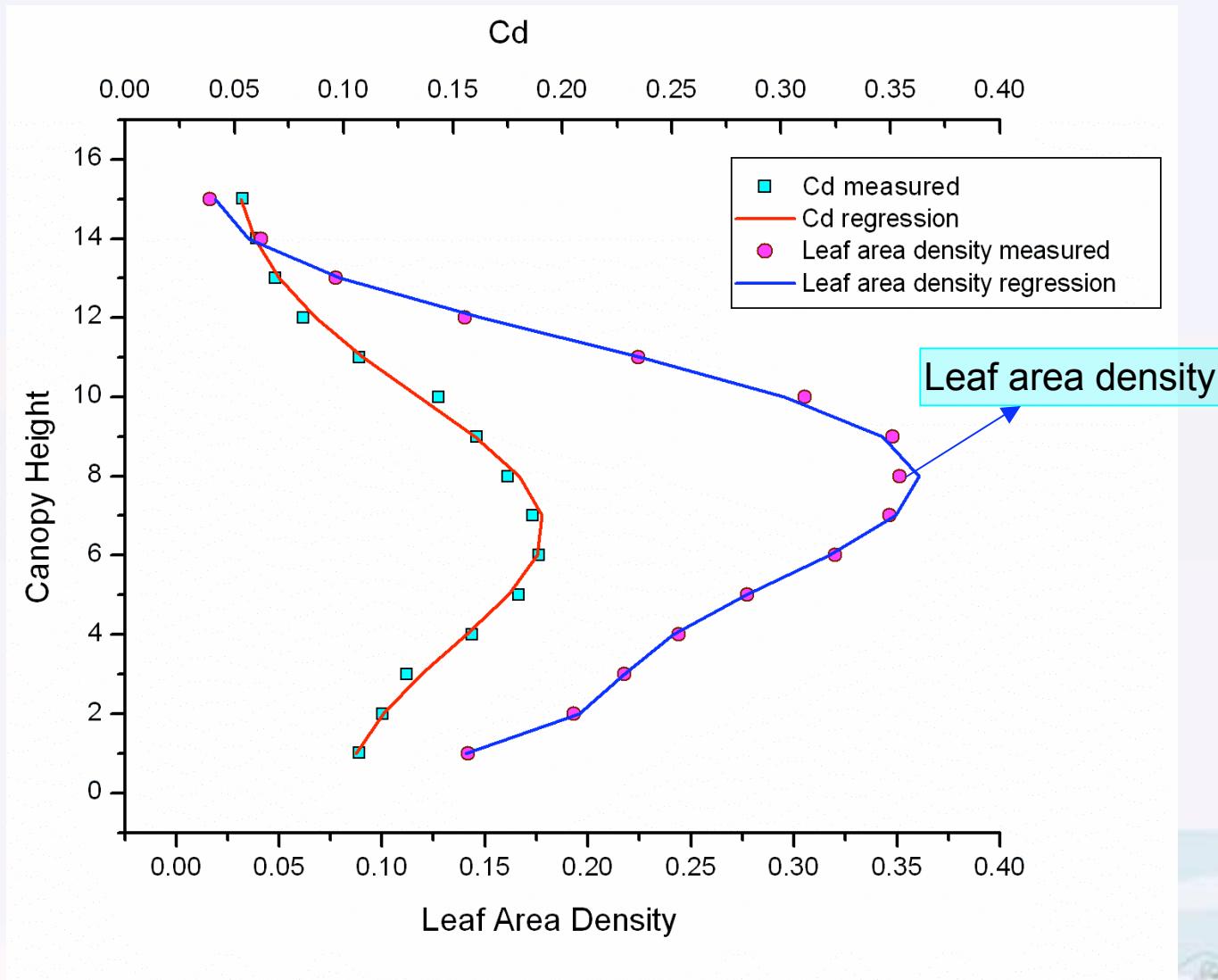
$$\begin{cases} \bar{u}(0)_{in-canopy} = \bar{u}(0)_{above-canopy} \\ \Delta\theta|_{n=0,in-canopy} = \Delta\theta|_{n=0,above-canopy} = \Delta\theta_s \\ \frac{d\bar{u}}{dn}|_{n=0,in-canopy} = \frac{d\bar{u}}{dn}|_{n=0,above-canopy} \end{cases}$$



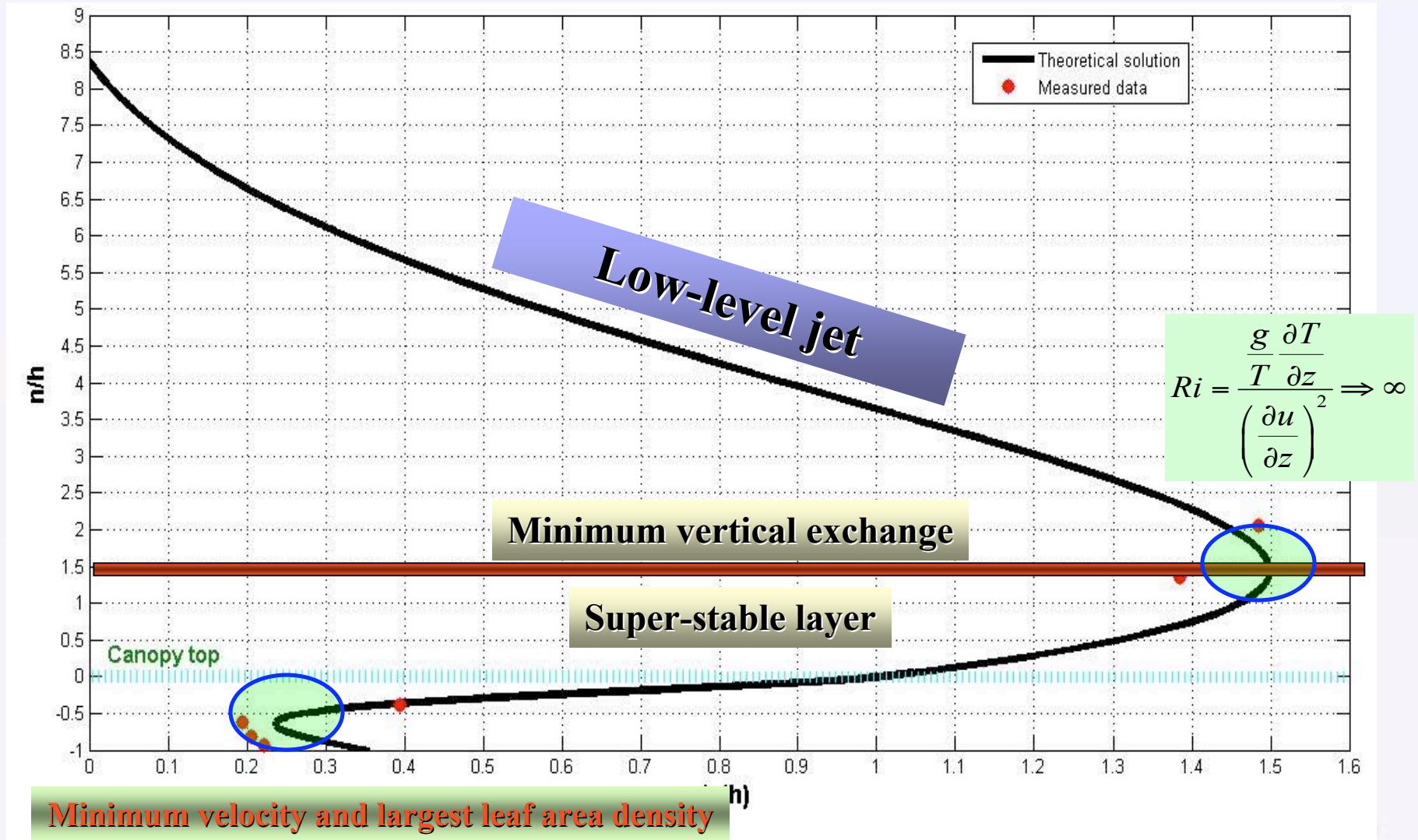
$$\begin{cases} U(n) = -\left(\frac{c_D(0)}{c_D(n)}K^2C^2e^{-[LAI-L(n)]} - \frac{g\beta\Delta\theta_s \sin\alpha}{c_D(n)} \int_n^0 e^{-[L(n)-L(n')]dn'}\right)^{1/2} & -h \leq n \leq 0 \\ U(n) = Ke^{-n/l}[\Delta\theta_s \sin(n/l) - C \cos(n/l)] & n \geq 0 \end{cases}$$



# Validation and Discussion



# Velocity Profile



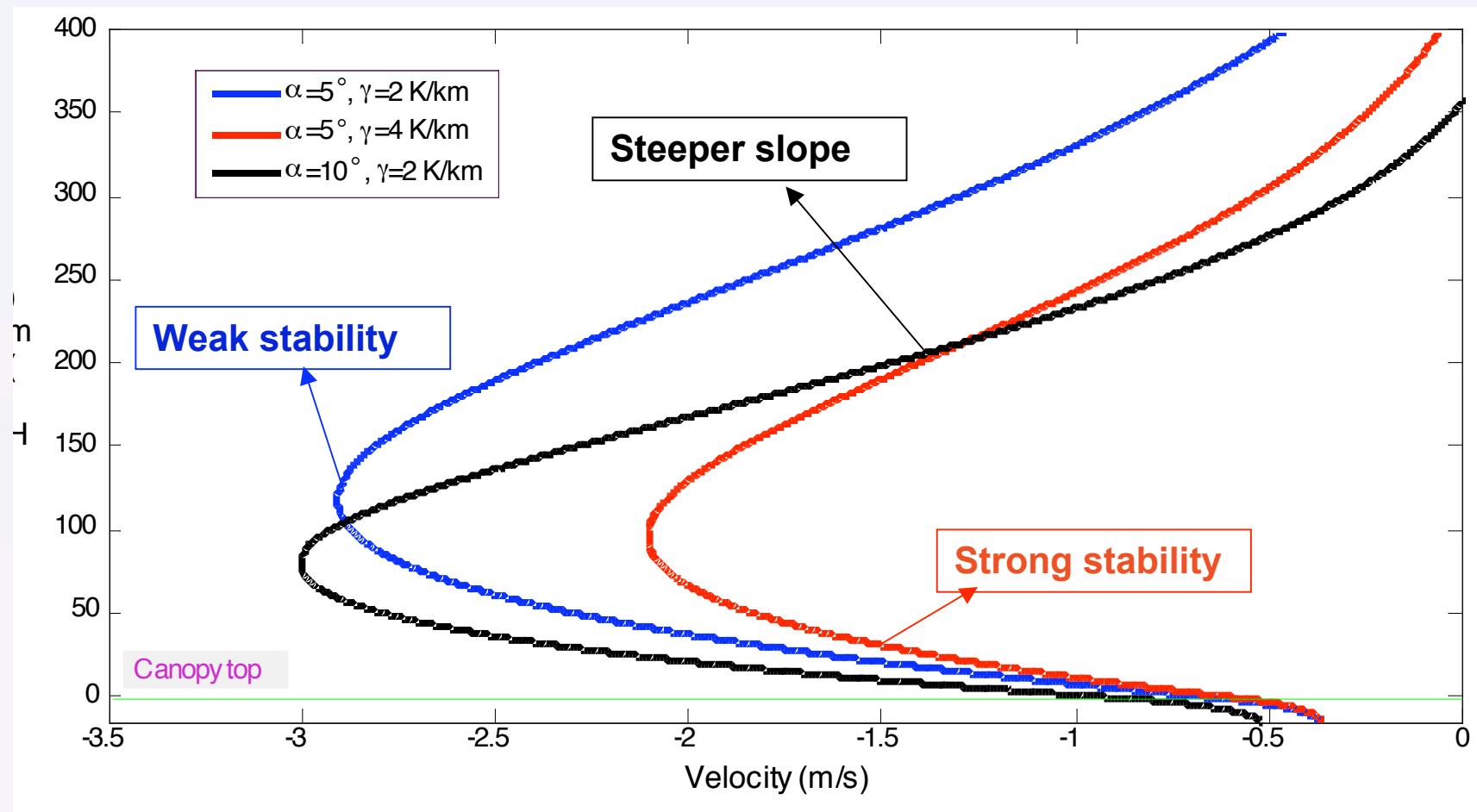
# Sensitive Study

## ---Uniform Case

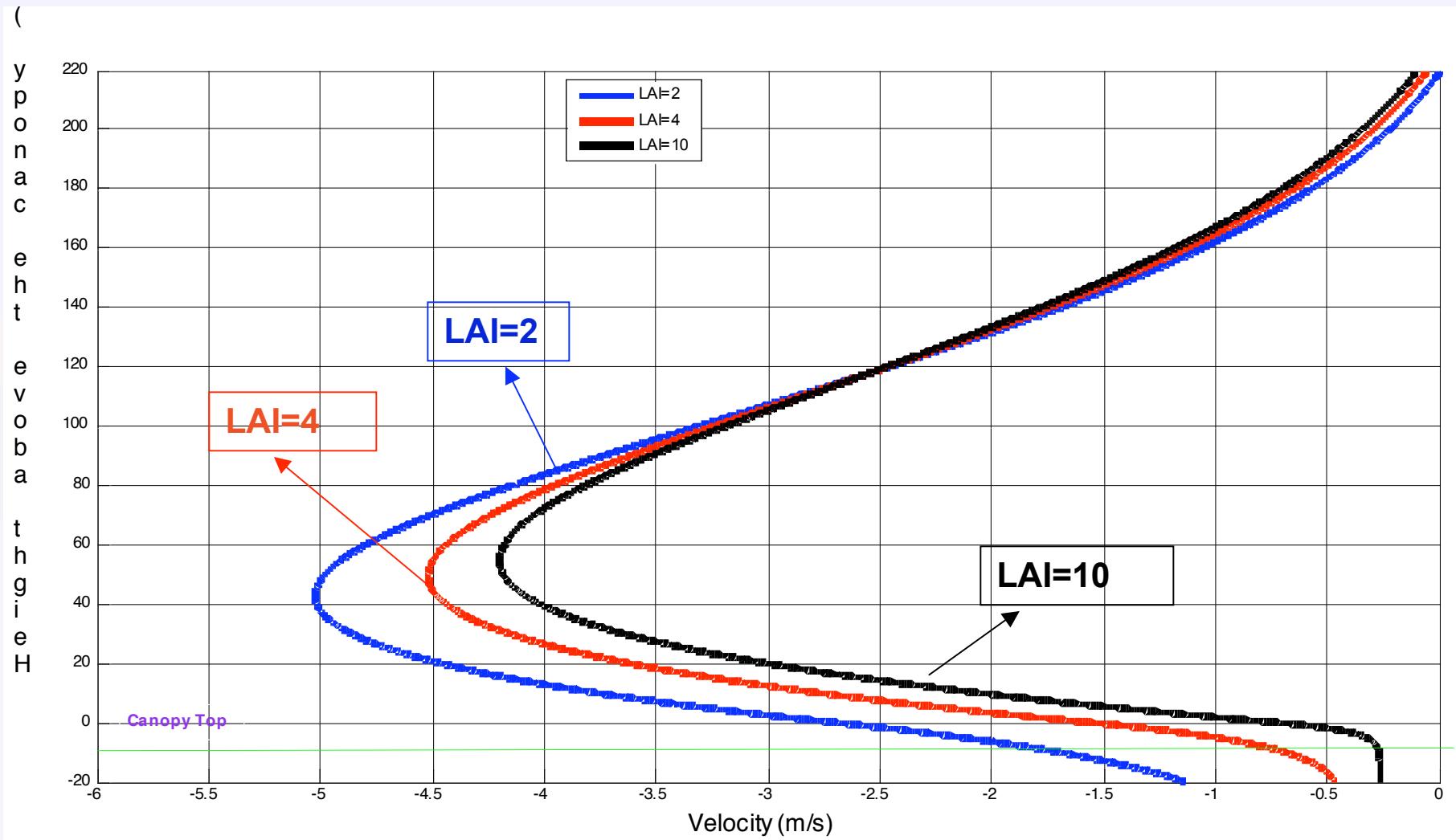
- Atmospheric Stability
- Slope angle
- Canopy morphology



# Influence of Atmospheric Stability and Slope Angle



# Influence of Plant Canopy



# Conclusions

- Analytical solution on cold-air-drainage winds by accounting for the influence of tree canopy is obtained.
- The effect of atmospheric stability and slope inclination is also investigated.
- The influence of different leaf area indexes is studied.





# Thank you!

